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OPTIMIZATION OF DYNAMIC PLASTIC DEFORMATION OF PLATES WITH A COMPLEX CONTOUR

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A method for studying dynamic deformation of ideal rigid-plastic plates with a complex contour on a viscoelastic foundation is proposed. The method allows one to optimize the process of pulsed forming. The optimization parameters are the amplitude of the pulsed load, viscoelastic damping coefficients of the foundation, the surface density of the plate material, and the shape and supporting conditions of the edges. Numerical examples of simply- and doubly-connected plates are given. It is shown that different combinations of the control parameters of the process can ensure the same final shape of the plate formed.

In connection with the development and use of pulse methods for metal treatment [1–4], it is of interest to study the mechanisms of deformation of a blank into an article and determine the dependence of the final parameters of the article on the shape of the blank and external action.

The process of deformation depends on many factors, in particular, on the properties of the blank material, the supporting conditions of its boundaries, the properties of the damping foundation, and the character of dynamic loading. Various shapes of the final deflection of the plate can be obtained by varying the supporting conditions at its edges for an unchanged distribution of the surface load.

In the present paper, a method of studying dynamic deformation of rigid-plastic plates with a complex contour on a viscoelastic foundation is proposed. The problem is solved in two stages. In the first stage, the direct problem is solved: the final deflection of the plate on an arbitrary viscoelastic foundation under pulsed loading is determined. In the second stage, the inverse problem is solved: for a given final deflection, the optimal parameters of deformation are found.

We consider an ideal rigid-plastic plate with a complex contour under an arbitrary dynamic pulsed load of intensity $P(t)$ uniformly distributed over the plate surface. The plate contour may be a circle, a regular polygon, a regular polygon with rounded corners or a piecewise smooth curve formed from the latter by changing the relative position of its circular and linear segments, and an irregular polygon into which a circle can be inscribed. The plate is assumed to be clamped or simply supported. The dynamic behavior of the above-mentioned plates is similar and studied in detail in [5, 6]. We consider the following doubly-connected plates: a regular polygonal plate, an annular plate, a regular polygonal plate with rounded corners or a plate formed from the latter by changing the relative position of circular and linear segments of the contour, and an irregular polygonal plate with a contour into which a circle can be inscribed. The external and internal contours of the plate are simply supported or clamped. The dynamic behavior of these doubly-connected plates is similar [7, 8].

Nemirovsky and Romanova [5–8] derived general equations of motion of the plates. In the present paper, we study the effect of a viscoelastic foundation on the process of plastic dynamic deformation.

For sufficiently intense loads, the dynamics of plates may be accompanied by the appearance, development, and disappearance of a zone of intense plastic deformation $I_{\rm pl}$, which moves translationally.

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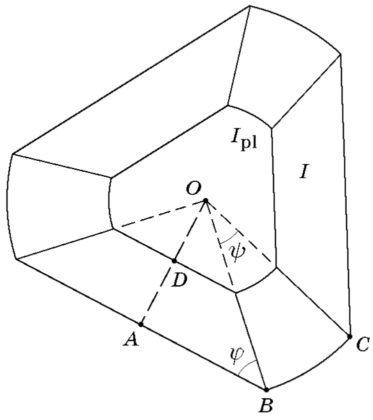

Fig. 1

1. DIRECT PROBLEM

Simply-Connected Plates. Figure 1 shows a regular polygonal plate with rounded corners ($|AO| = r$, $|AD| = \delta r$, $\angle OBA = \varphi$, $\angle BOC = \psi$, and $\angle OAB = \pi/2$, where r is the radius of the circle inscribed into the polygonal contour or the radius of a circular plate). The values $\varphi = \pi/2$ and $\psi = 0$ refer to circular and regular polygonal plates, respectively. The equations governing the motion of these plates have the form [5, 6]

$$\delta^3(4 - 3\delta)(\ddot{\alpha} + k_2\dot{\alpha} + k_1\alpha) = 2p_1\delta^2(3 - 2\delta) - m_0, \tag{1.1}$$

$$(\delta\dot{\alpha})^{\cdot} + \delta(k_2\dot{\alpha} + k_1\alpha) = p_1, \tag{1.2}$$

where $p_1 = P/r$, $m_0 = M(2 - \eta)$ for regular polygonal plates, circular plates, and irregular plates with a contour into which a circle can be inscribed, $m_0 = M(\cot\varphi + \psi)(2 - \eta)/(\cot\varphi + \psi/\sin^2\varphi)$ for polygonal plates with rounded corners, $M = 12M_0t_0^2/(\rho r^3)$, ρ is the surface mass density of the plate material, M_0 is the limit plastic bending moment, $\eta = 0$ for the clamped contour and $\eta = 1$ for the simply supported contour, α is the angle of rotation of the rigid region I about the supporting edge, $\delta(\tau)$ is a dimensionless function characterizing the size of the central plastic region $I_{\rm pl}$, the dot denotes differentiation with respect to the parameter τ ($\tau = t/t_0$), t is the current time, t_0 is the characteristic time, $k_1 = K_1^0t_0^2/\rho$ and $k_2 = K_2^0t_0^2/\rho$, and K_1^0 and K_2^0 are the coefficients of elastic and viscous resistance of the foundation.

Nemirovsky and Romanova [9] showed that an arbitrary load $p_1(\tau)$ can be replaced by an equivalent constant load. By virtue of this inference, we consider, for simplicity, the rectangular loading pulse

$$p_1(\tau) = p_1 = \text{const}, \quad 0 \leqslant \tau \leqslant 1, \qquad p_1(\tau) = 0, \quad \tau > 1. \tag{1.3}$$

The initial conditions for α have the form

$$\alpha(0) = \dot{\alpha}(0) = 0. \tag{1.4}$$

To determine the quantity δ_0 in the initial condition

$$\delta(0) = \delta_0, \tag{1.5}$$

it is necessary to solve a supplementary problem considered below. The Cauchy problem (1.1), (1.2), (1.4), and (1.5) satisfy the existence and uniqueness theorem [10]; therefore, the quantity δ remains constant during loading: $\delta(\tau) = \delta(0) = \delta_0$.

Let $p_1 > p_1^0$, where $p_1^0 = m_0/2$ is the limit plastic pressure determined in [6]. Then, Eq. (1.2) implies the equality $\delta(\ddot{\alpha} + k_2\dot{\alpha} + k_1\alpha) = p_1$. With allowance for this equality, from Eq. (1.1) we obtain

$$\delta^2(2 - \delta) = m_0/p_1 = 2p_1^0/p_1. \tag{1.6}$$

It follows from (1.6) that $\delta < 1$ for $p_1 > 2p_1^0$ and $\delta \geqslant 1$ for $p_1 \leqslant 2p_1^0$. Thus, for the load $p_1 > 2p_1^0$ ("high" loads), the plate motion occurs owing to the presence of a plastic zone and is governed by system (1.1), (1.2) subject to the initial conditions (1.4) and (1.5), where δ_0 is determined from (1.6) and does not depend on the resistance of the foundation. For the load $p_1^0 < p_1 \leqslant 2p_1^0$ ("moderate" loads), since δ cannot be greater than unity, the plate motion occurs in the absence of a plastic zone and is governed by Eq. (1.1) for $\delta = 1$ and the initial conditions (1.4). Let us consider both types of loading.

"Moderate" Load ($p_1^0 < p_1 \leqslant 2p_1^0$). In the first phase ($0 \leqslant \tau \leqslant 1$, $p_1 = \text{const} > 0$, and $\delta \equiv 1$), the motion is described by the equation $\ddot{\alpha} + k_2\dot{\alpha} + k_1\alpha = 2(p_1 - p_1^0)$ subject to the initial conditions (1.4).

The second phase ($1 < \tau \leqslant \tau_1$) is the plate motion from the moment the load is removed to the moment the plate ceases to move. In this case, $\delta \equiv 1$ and $p_1 = 0$, and the behavior of the plate is described by the equation $\ddot{\alpha} + k_2\dot{\alpha} + k_1\alpha = -2p_1^0$ for which the initial conditions $\alpha(1)$ and $\dot{\alpha}(1)$ are determined in the end of the first phase. The time τ_1 is found from the condition $\dot{\alpha}(\tau_1) = 0$. The deflections at the point (x, y) are calculated by the formula $w(x, y, \tau) = \alpha(\tau)d(x, y)/r$, where $d(x, y)$ is the distance from the point (x, y) to the supporting edge I and $w = W/r$ (W is the deflection of the plate). The final deflection at the center of the plate w_f is calculated by the formulas

$$w_f = A - B\exp(-k_2T) - 2p_1^0Tk_2^{-1} \quad \text{for} \quad k_1 = 0, \quad k_2 \neq 0, \tag{1.7}$$

where $T = -\ln[2p_1^0/(Bk_2^2)]/k_2$, $A = 2(p_1^0/k_2 + p_1)/k_2$, and $B = 2[p_1(\exp(k_2) - 1) + p_1^0]/k_2^2$;

$$w_f = A\exp(\lambda_1T) + B\exp(\lambda_2T) - 2p_1^0/k_1 \quad \text{for} \quad k_1 \neq 0, \quad k_2 \neq 4k_1, \tag{1.8}$$

where $\lambda_{1,2} = \left(-k_2 \pm \sqrt{k_2^2 - 4k_1}\right)\Big/2$, $A = 2\lambda_2[-p_1 + p_1^0 + p_1\exp(-\lambda_1)][k_1(\lambda_2 - \lambda_1)]^{-1}$, $B = 2\lambda_1[-p_1 + p_1^0 + p_1\exp(-\lambda_2)][k_1(\lambda_1 - \lambda_2)]^{-1}$, and $T = \ln[-B\lambda_2/(A\lambda_1)]/(\lambda_1 - \lambda_2)$;

$$w_f = (A + BT)\exp(\lambda_3T) - 2p_1^0 \quad \text{for} \quad k_1 \neq 0, \quad k_2 = 4k_1, \tag{1.9}$$

where $\lambda_3 = -k_2/2$, $A = -2(p_1 - p_1^0)$, $B = 2\lambda_3[p_1 - p_1^0 + p_1\exp(-\lambda_3)]$, and $T = -(A\lambda_3 + B)/\lambda_3$;

$$w_f = A\cos(\lambda_4T) + B\sin(\lambda_4T) - 2p_1^0/k_1 \quad \text{for} \quad k_1 \neq 0, \quad k_2 = 0, \tag{1.10}$$

where $\lambda_4 = \sqrt{k_1}$, $A = -2(p_1 - p_1^0 + p_1\cos\lambda_4)/k_1$, $B = 2p_1\sin\lambda_4/k_1$, and $T = \arctan(B/A)/\lambda_4$;

$$w_f = p_1^2(1 - p_1^0/p_1)/p_1^0 \quad \text{for} \quad k_1 = k_2 = 0. \tag{1.11}$$

"High" Load ($p_1 > 2p_1^0$). In the first phase ($0 \leqslant \tau \leqslant 1$ and $p_1 = \text{const} > 0$), the motion is described by Eq. (1.1) under the initial conditions (1.4) and (1.5), where $\delta = \delta_0$ is determined from (1.6).

In the second phase ($1 < \tau \leqslant \tau_1$ and $p_1 = 0$), the motion is described by the system

$$\delta^3(4 - 3\delta)(\ddot{\alpha} + k_2\dot{\alpha} + k_1\alpha) = -2p_1^0, \qquad (\delta\dot{\alpha})^{\cdot} + \delta(k_2\dot{\alpha} + k_1\alpha) = 0, \tag{1.12}$$

which implies $\dot{\delta} = 2p_1^0/[\dot{\alpha}\delta^2(4 - 3\delta)] > 0$. In this phase, the function $\delta(\tau)$ increases. The time τ_1 is determined from the condition $\delta(\tau_1) = 1$ and corresponds to the moment when the plastic zone shrinks to a point.

In the third phase ($\tau_1 < \tau \leqslant \tau_2$, $p_1 = 0$, and $\delta \equiv 1$), the motion is described by Eq. (1.1) with $\delta = 1$ up to the moment the plate ceases to move at the moment τ_2, which is determined from the condition $\dot{\alpha}(\tau_2) = 0$.

The deflections are determined from the equation $\dot{w}(x, y, \tau) = d(x, y)\dot{\alpha}(\tau)/r$ at the points $(x, y) \in I$ and from the equation $\dot{w}(x, y, \tau) = \delta(\tau)\dot{\alpha}(\tau)$ at the points $(x, y) \in I_{\text{pl}}$.

In all phases of motion, except for the second phase for the case of a "high" load, the equations of motion admit analytical solutions. In the second phase of the "high" load, system (1.12) is reduced to the form

$$\dot{\alpha} = \nu, \quad \dot{\nu} = -2p_1^0/[\delta^3(4 - 3\delta)] - k_2\nu - k_1\alpha, \quad \dot{\delta} = 2p_1^0/[\nu\delta^2(4 - 3\delta)]$$

and solved numerically by the Runge–Kutta method.

With the use of the method proposed, the motion of various simply-connected plates in a viscoelastic medium was studied. The calculation results suggest the following conclusions.

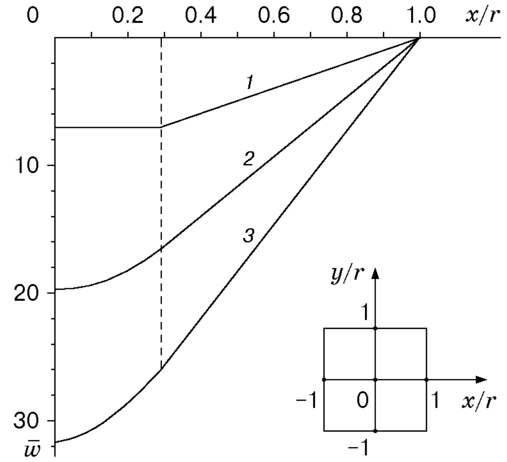

Fig. 2

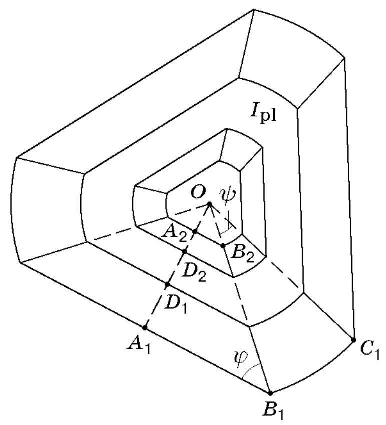

Fig. 3

1. Viscoelastic resistance of the medium has no effect on the mechanism of plate motion. The motion of a plate on a viscoelastic foundation includes the same phases as the motion of a plate without resistance of the medium [5, 6]. For sufficiently high loads, the dynamics of plates on a viscoelastic foundation, as in the case without resistance of the medium, is accompanied by the appearance, development, and disappearance of a zone of intense plastic deformation.

2. Viscoelastic resistance of the medium reduces final deflections and the duration of motion of the plate and affects substantially the shape of final deflections. Figure 2 shows the curves of final deflections in the section $y = 0$ of a square simply supported plate for $p_1 = 3p_1^0$ [$\bar{w} = Wr^2\rho/(M_0t_0^2)$]; curve 1 refers to $k_1 = k_2 = 0$, curve 2 to $k_1 = 0.5$ and $k_2 = 0$, and curve 3 to $k_1 = 3$ and $k_2 = 1$.

3. For $p_1/p_1^0 = \text{const}$, if p_1^0 varies m-fold, the final deflection varies m-fold too. This fact allows one to use a square simply supported plate as a certain model structure for calculation of all simply-connected plates under consideration in a damping medium, since the character of supports and the number of edges influence only the limit plastic pressure.

Doubly-Connected Plates. Figure 3 shows a doubly-connected regular polygonal plate with rounded corners ($|OA_1| = r$, $|A_1D_1| = \delta r$, $|A_1D_2| = \xi r$, $|A_1A_2| = \lambda r$, $\angle OA_1B_1 = 90°$, $\angle OB_1A_1 = \varphi$, and $\angle B_1OC_1 = \psi$). The cases $\varphi = \pi/2$ and $\psi = 0$ correspond to annular and doubly-connected regular polygonal plates, respectively. The equations that describe the motion of these plates have the form [7, 8]

$$\delta\dot{\alpha}_1 = (\lambda - \xi)\dot{\alpha}_2; \qquad (1.13)$$

$$\delta^3(4 - 3\delta)(\ddot{\alpha}_1 + k_2\dot{\alpha}_1 + k_1\alpha_1) = 2p_1\delta^2(3 - 2\delta) - m_1; \qquad (1.14)$$

$$(\lambda - \xi)^3(4 - 3\xi - \lambda)(\ddot{\alpha}_2 + k_2\dot{\alpha}_2 + k_1\alpha_2) = 2p_1(\lambda - \xi)^2(3 - 2\xi - \lambda) - m_2; \qquad (1.15)$$

$$(\delta\dot{\alpha}_1)^{\cdot} = p_1 - \delta(k_2\dot{\alpha}_1 + k_1\alpha_1). \qquad (1.16)$$

In the case where I_{pl} degenerates, Eq. (1.16) is replaced by the condition

$$\delta = \xi. \qquad (1.17)$$

In (1.13)–(1.18), $m_j = 12M(2 - \eta_j)(1 - \lambda_j)$ ($j = 1, 2$) for doubly-connected regular polygonal plates, annular plates, and irregular plates with a contour into which a circle can be inscribed, $m_j = 12M(1 - \lambda_j)[\cot\ \varphi(2 - \eta_j) + \psi(2 - \theta_j)]/(\cot\varphi + \psi/\sin^2\varphi)$ ($j = 1, 2$) for plates with rounded corners, $\lambda_1 = 0$, $\lambda_2 = \lambda$, $M = 12M_0t_0^2/(\rho r^3)$, $\eta_j = 0$ and $\theta_j = 0$ for a clamped contour, $\eta_j = 1$ and $\theta_j = 1$ for a simply supported contour, $j = 1$ and 2 refer to the external and internal contours, respectively, α_j is the angle of rotation of the rigid region I_j about the supporting edge, λ is a dimensionless quantity characterizing the size of the hole in the plate, and $\delta(\tau)$ and $\xi(\tau)$ are dimensionless functions determining the size of the internal plastic region $I_{\rm pl}$.

As in the case of simply-connected plates, we consider a rectangular loading pulse (1.3) for simplicity.

The initial conditions for α_i and $\dot\alpha_i$ have the form

$$\dot\alpha_i(0) = \alpha_i(0) = 0 \qquad (i = 1,\ 2). \tag{1.18}$$

To determine the quantities $\delta_{\rm in}$ and $\xi_{\rm in}$ in the initial conditions

$$\delta(0) = \delta_{\rm in}, \qquad \xi(0) = \xi_{\rm in}, \tag{1.19}$$

it is necessary to solve two supplementary problems considered below.

For a constant load, one can integrate system (1.13)–(1.17) assuming that δ and ξ are constant. Since the Cauchy problem (1.14)–(1.18) satisfies the conditions of the existence and uniqueness theorem [10], the quantities δ and ξ remain constant during loading: $\delta = \delta(0) = \delta_{\rm in}$ and $\xi = \xi(0) = \xi_{\rm in}$.

Let $p_1 > p_0$, where p_0 is the limit plastic pressure determined in [7, 8]: $p_0 = m_1/[2\delta_0^2(3-2\delta_0)]$ and $m_1(\lambda-\delta_0)^2(3-2\delta_0-\lambda) = m_2\delta_0^2(3-2\delta_0)$. Then, Eqs. (1.13) and (1.16) imply $\delta(\ddot\alpha_1 + k_2\dot\alpha_1 + k_1\alpha_1) = p_1$ and $(\lambda-\xi)(\ddot\alpha_2 + k_2\dot\alpha_2 + k_1\alpha_2) = p_1$. With allowance for these equalities, from (1.14) and (1.15) we obtain

$$\delta^2(2-\delta) = m_1/p_1, \qquad (\lambda-\xi)^2(2-\lambda-\xi) = m_2/p_1. \tag{1.20}$$

It follows from (1.20) that $\xi > \delta_* > \delta$ for $p_1 > p_*$ and $\xi \leqslant \delta$ for $p_1 \leqslant p_*$, where p_* and δ_* are determined from the equations $m_1(\lambda-\delta_*)^2(2-\delta_*-\lambda) = m_2\delta_*^2(2-\delta_*)$ and $p_* = m_1/[\delta_*^2(2-\delta_*)]$. Thus, for the load $p_1 > p_*$ ("high" load), the plate motion occurs in the presence of a developed plastic zone $I_{\rm pl}$ and is described by system (1.13)–(1.16) for the initial conditions (1.18) and (1.19), where $\xi_{\rm in}$ and $\delta_{\rm in}$ are determined from (1.20). For the load $p_0 < p_1 \leqslant p_*$ ("moderate" loads), since $\delta \leqslant \xi$, the plate motion occurs in the absence of a plastic zone $I_{\rm pl}$ and is described by system (1.13)–(1.15), (1.17) for the initial conditions (1.18) and (1.19), in which $\xi_{\rm in} = \delta_{\rm in}$. In this case, $\delta_{\rm in}$ is determined in the following manner. It follows from (1.13) and (1.17) that $\delta(\ddot\alpha_1 + k_2\dot\alpha_1 + k_1\alpha_1) = (\lambda-\xi)(\ddot\alpha_2 + k_2\dot\alpha_2 + k_1\alpha_2)$. From the last equality and (1.14) and (1.15), we obtain the equation for $\delta_{\rm in}$:

$$\frac{2p_1\delta_{\rm in}^2(3-2\delta_{\rm in}) - m_1}{\delta_{\rm in}^2(4-3\delta_{\rm in})} = \frac{2p_1(\lambda-\delta_{\rm in}^2)(3-2\delta_{\rm in}^2-\lambda) - m_2}{(\lambda-\delta_{\rm in}^2)(4-3\delta_{\rm in}-\lambda)}. \tag{1.21}$$

Let us consider both loading cases in more detail.

"Moderate" Load ($p_0 < p_1 \leqslant p_*$). In the first phase ($0 \leqslant \tau \leqslant 1$ and $p_1 = \text{const} > 0$), the motion is described by system (1.14), (1.15), (1.18), (1.19), in which $\delta_{\rm in}$ is determined from (1.21). The first phase ends when the load is removed.

In the second phase ($1 < \tau \leqslant \tau_1$ and $p_1 = 0$), inertial motion occurs up to the moment τ_1 and is described by system (1.13), (1.17) and

$$\delta^3(4-3\delta)(\ddot\alpha_1 + k_2\dot\alpha_1 + k_1\alpha_1) = -m_1, \quad (\lambda-\xi)^3(4-3\xi-\lambda)(\ddot\alpha_2 + k_2\dot\alpha_2 + k_1\alpha_2) = -m_2. \tag{1.22}$$

This system is solved numerically by the Runge–Kutta method. The time τ_1 is determined from the condition $\dot\alpha_i(\tau_1) = 0$ ($i = 1, 2$). The calculation yields $\dot\delta(\tau_1) = 0$.

"High" Load ($p_1 > p_*$). In the first phase ($0 \leqslant \tau \leqslant 1$ and $p_1 = \text{const} > 0$), the motion is described by system (1.13), (1.14), where $\delta_{\rm in}$ and $\xi_{\rm in}$ are determined from (1.20). The first phase ends when the load is removed.

In the second stage ($1 < \tau \leqslant \tau_1$ and $p_1 = 0$), the motion is described by system (1.13), (1.22) and $(\delta\dot\alpha_1)^{\cdot} = -\delta(k_2\dot\alpha_1 + k_1\dot\alpha_1)$, which implies $\dot\delta = m_1/[\dot\alpha_1\delta^2(3-2\delta)] > 0$ and $\dot\xi = -\,m_2/[\dot\alpha_2(\lambda-\xi)^2(4-3\xi-\lambda)] < 0$. Thus, the function $\delta(\tau)$ increases and the function $\xi(\tau)$ decreases. The moment τ_1 is determined from the condition $\delta(\tau_1) = \xi(\tau_1)$, which corresponds to complete shrinkage of the zone $I_{\rm pl}$ into a linear segment. System (1.13), (1.22) is solved numerically by the Runge–Kutta method.

The third phase ($\tau_1 < \tau \leqslant \tau_2$) is the motion of the plate up to its halt, which is described by system (1.13), (1.17), (1.22) solved numerically. The moment τ_2 is determined from the condition $\dot\alpha_i(\tau_2) = 0$ ($i = 1$, 2). The calculations show that $\dot\delta(\tau_2) = 0$.

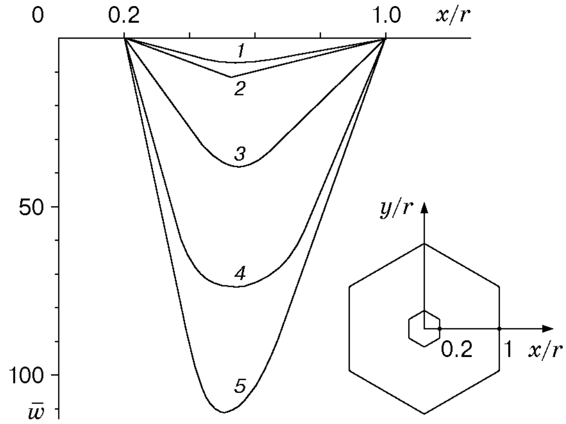

Fig. 4

In all the phases, the deflections at the points $(x, y) \in I_i$ $(i = 1,\ 2)$ are calculated by the formula $w(\tau) = \dfrac{d_i(x,y)}{r} \displaystyle\int_{\tau_k}^{\tau} \delta(s)\dot{\alpha}_1(s)\,ds + w(\tau_k)$, where $d_i(x, y)$ is the distance from the point (x, y) to the supporting edge I_i and τ_k is the time of phase beginning. At the points $(x, y) \in I_{\rm pl}$, the deflections are determined from the equation $\dot{w} = \delta\dot{\alpha}_1$.

Figure 4 shows the curves of final deflections in the cross section $y = 0$, which occur in a simply supported regular polygonal doubly-connected plate with $\lambda = 0.8$ in a damping medium ($P_1 = Pr^2/M_0$) (curve 1 refers to $P_1 = 50.55$, $k_1 = 1$, and $k_2 = 5$, curve 2 to $P_1 = 17.7$, $k_1 = 0$, and $k_2 = 0.5$, curve 3 to $P_1 = 50.55$ and $k_1 = k_2 = 0.5$, curve 4 to $P_1 = 90$ and $k_1 = k_2 = 0.5$, and curve 5 to $P_1 = 50.55$ and $k_1 = k_2 = 0$).

2. INVERSE PROBLEM

As is noted above, viscoelastic resistance substantially influences the shape of final deflections. Varying the load magnitude and the damping coefficients, one can change the shape of final deflections within a wide range. The final deflection depends also on the limit pressure p_1^0 and, hence, on the shape and supporting conditions of the plate edges. One can determine the function U that establishes a correspondence between each set $\boldsymbol{f} = (p_1^0, p_1, k_1, k_2)$ and a certain characteristic of the final shape Π: $U(\boldsymbol{f}) = \Pi$.

The characteristic Π may include many determining quantities: final deflections at any point of the plate, slopes of the deformed surface of the plate relative to the horizontal line at the fixed points of the plate, surface area and filling volume of the deformed plate, etc. For example, if it is necessary to form an article with a specified filling volume, the characteristic Π should be $\Pi = V$, where $V = \displaystyle\int_0^{t_f} \iint_S \dot{w}(\tau, s)\,ds\,d\tau$ (t_f is the time of plate deformation, S is the surface, and ds is the surface element). If it is required to obtain the maximum or minimum filling volume, Π is taken to be $\Pi = \max V$ or $\Pi = \min V$, respectively.

All quantities that enter the characteristic Π are determined by the properties of the article formed. Varying the set of quantities $\boldsymbol{f}$, one can obtain different shapes of final deflections.

The inverse problem is formulated as follows: for a given characteristic Π, it is required to find the quantities $\boldsymbol{f}$:

$$\boldsymbol{f} = U^{-1}(\Pi). \tag{2.1}$$

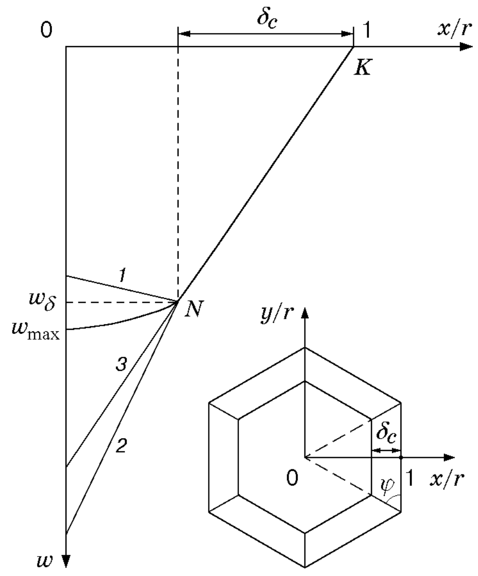

Fig. 5

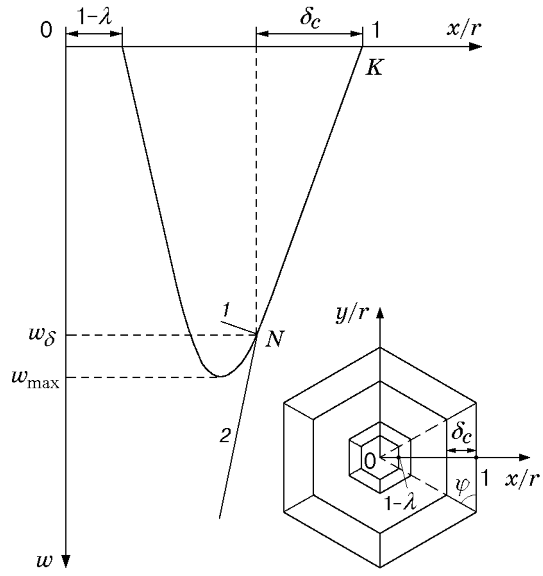

Fig. 6

Thus, it is possible to control the deformation processes. Namely, for a given characteristic Π, one can determine the set $\boldsymbol{f}$ that ensures the required shape of the article. These problems are solved numerically using computers.

The inverse problem (2.1) is solved by the local-variation method [11]. As an example of the solution of the inverse problem (2.1), we consider the following problem. The limit plastic pressure p_1^0 is assumed to be fixed. The required shape of the deflection is determined by the following three characteristic parameters: the maximum deflection $w_{\max}$, the deflection w_δ, and the quantity $0 < \delta_c < 1$ for simply-connected plates (Fig. 5) and $0 < \delta_c \leqslant \delta_0 < 1$ for doubly-connected plates (Fig. 6). The quantity δ_c is chosen in such a way that the segment KN in Figs. 5 and 6 is a linear segment. Moreover, if $w_\delta = \delta_c w_{\max}$ for simply-connected plates, "moderate" loads are considered and k_1 and k_2 are determined from Eqs. (1.7)–(1.11). If $\delta_c w_{\max} < w_\delta \leqslant w_{\max}$, the amplitude of the rectangular loading pulse p_1 is uniquely determined for the quantity δ_c from Eq. (1.6). The cases $w_\delta > w_{\max}$ and $\delta_c w_{\max} > w_\delta$ are not considered, since these shapes of the final deflection cannot be obtained within the framework of the model used. These cases are shown in Fig. 5 (deflection shape 1 refers to $w_\delta > w_{\max}$, shape 2 to $\delta_c w_{\max} > w_\delta$, and shape 3 to $w_\delta = \delta_c w_{\max}$). For doubly-connected plates with $\delta_c w_{\max}/\lambda < w_\delta \leqslant w_{\max}$, the amplitude p_1 is determined from the first equation in (1.20). Figure 6 shows the final deflections that cannot be obtained within the framework of the model considered (deflection shape 1 refers to the case $w_\delta > w_{\max}$ and shape 2 to the case $\delta_c w_{\max}/\lambda < w_\delta$).

It should be noted that, for identical loads, the final deflection determined with allowance for resistance of the foundation must not exceed the final deflection determined at the same points without allowance for resistance. This condition can be satisfied by increasing the loading time t_0.

The solution of the inverse problem by the local-variation method depends on the initial approximations [11]; therefore, the solution of problem (2.1) is not unique. The same final deflection can be obtained for different resistances of the foundation. For example, for a simply supported plate, the values $\bar{w}_\delta = 4.4$, $\bar{w}_{\max} = 4.4$, and $\delta_c = 0.72$ correspond to the following combinations of parameters: $P_1 = 18$, $k_1 = 8$, and $k_2 = 0$ and $P_1 = 18$, $k_1 = 1$, and $k_2 = 3$; for a simply supported doubly-connected plate with $\lambda = 0.8$, the values $\bar{w}_\delta = 7$, $\bar{w}_{\max} = 8$, and $\delta_c = 0.383$ correspond to the combinations $P_1 = 50.59$, $k_1 = 5.9$, and $k_2 = 3$ and $P_1 = 50.59$, $k_1 = 1.8$, and $k_2 = 5$.

The nonuniqueness of the solution of the inverse problem (2.1) allows one to choose the parameters of resistance of the foundation that ensure the required final deflection of the plate.

REFERENCES

1. *Pulsed Plastic Metal Working* [in Russian], Mashinostroenie, Moscow (1977).
2. W. Johnson, "Review of metal working plasticity," *Metallurg. Metal Form.*, **39**, No. 5, 147–151 (1972).
3. W. Johnson, A. Poynton, H. Singh, and F. M. Travis, "Experiments in the underwater explosive stretch forming of clamped circular blanks," *Int. J. Mech. Sci.*, **8**, 237–270 (1966).
4. J. S. Rinehart and J. Pearson, *Behavior of Metals under Impulsive Loads*, The American Society for Metals, Cleveland, Ohio (1954).
5. Yu. V. Nemirovsky and T. P. Romanova, "Dynamic bending of polygonal plastic plates," *Prikl. Mekh. Tekh. Fiz.*, No. 4, 149–157 (1988).
6. Yu. V. Nemirovsky and T. P. Romanova, "Dynamics of polygonal plastic plates with rounded corners," *Probl. Prochn.*, No. 9, 62–66 (1991).
7. Yu. V. Nemirovsky and T. P. Romanova, "Dynamic behavior of doubly-connected polygonal plastic plates," *Prikl. Mekh.*, **23**, No. 5, 53–59 (1987).
8. Yu. V. Nemirovsky and T. P. Romanova, "Dynamics of doubly-connected plastic plates with piecewise smooth supported contours," *Prikl. Mekh.*, **28**, No. 4, 24–31 (1992).
9. Yu. V. Nemirovsky and T. P. Romanova, "Effect of pulsed load form on final deflections of rigid-plastic plates of a complex shape," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 6, 113–121 (1995).
10. V. V. Stepanov, *Differential Equations* [in Russian], Fizmatgiz, Moscow (1959).
11. F. L. Chernous'ko and V. N. Banichuk, *Variational Problems of Mechanics and Control* [in Russian], Nauka, Moscow (1973).